4. *Conclusiones*

Hemos mostrado cómo el proceso de integración numérica de las ecuaciones de movimiento puede introducir en las ecuaciones de condición perturbaciones que pueden ser pequeñas pero no despreciables. Estas perturbaciones a su vez pueden amplificarse considerablemente en el proceso de corrección diferencial. Consideramos que ésta es una explicación plausible para las anomalías que se presentan en el cálculo de aquellos cometas que registran pasos muy próximos al Sol o a algún planeta. Las conclusiones que se obtengan acerca de las fuerzas no gravitatorias que actúan sobre los cometas no pueden considerarse como definitivas si no se ha hecho un análisis de los errores sistemáticos de cálculo para eliminarlos o al menos obtener una estimación correcta de ellos.

Nos proponemos realizar en el próximo futuro una serie sistemática de experimentos numéricos con órbitas y modelos de cometas típicos aplicando las técnicas resumidas en el presente informe.

(1) Marsden, B. G., A. J. 73, 1968.
Marsden, B. G., A. J. 74, 1969.
Marsden, B. G., A. J. 75, 1970.
(2) Pereyra, V., Siam Journal Num. Anal. 4, 1967.
(3) Pereyra, V. y Rosen, J. B., Stanford Un CS 13, 1964.
(4) Pereyra, V., Aequationes Mathematicas 2, 1969.
(5) Rosen J. B., J. Soc. Indust. Appl Math., 12, 1964.
(6) Zadunaisky, P. E., Proc. I.A.U. Symp. Nº 25, 1964.
(7) Zadunaisky, P. E., Proc. Symp. on "Periodic Orbits Resonance and Stability", S. Paulo, Brasil, 1969 (Reidel Publ. Co.).
(8) Zadunaisky, P. E., y Pereyra, V., Proc. Internat. Federation Inform. Processing Symposium, N. York, 1965, Vol. 2.

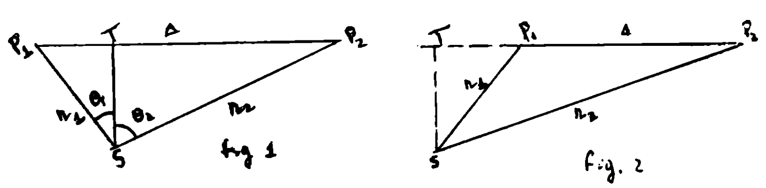
On a new form the main part of the disturbing function in the three-body problem

C. A. Altavista

Observatorio Astronómico, La Plata

Abstract: A new form for the disturbing function is given in terms of the heliocentric distances r_1 and r_2 and two auxiliary angles θ_1 and θ_2. An outline is given for obtaining expressions of these angles in terms of known quantities.

1. — Let be given three point mases $m_0(S)$, the Sun, $m_1(P_1)$ and $m_2(P_2)$ the planets. Let r_1 and $r_2 > r_1$, be the heliocentric distances of P_1 and P_2. Writing Δ for the mutual distance P_1P_2, and taking in figs. 1 and 2 ST perpendicular to P_1P_2



Figs. 1 and 2 show two possible configurations of the three-body problem, excluded the collinear case.

it is easily seen from triangles STP_1 and STP_2 that:

$$P_2T - P_1T = \frac{r_2^2 - r_1^2}{\Delta}$$

from which we obtain:

$$\frac{1}{\Delta} = \frac{1}{r_2^2} \, \frac{P_2T - P_1T}{1 - \dfrac{r_1^2}{r_2^2}}$$

It is evident that:

$$P_1T = r_1 \sin\theta_1 \quad , \quad P_2T = r_2 \sin\theta_2$$

2. — In order to put angles θ_1 and θ_2 in terms of orbital elements, let us take the projections of the orbits on the celestial sphere (fig. 3).

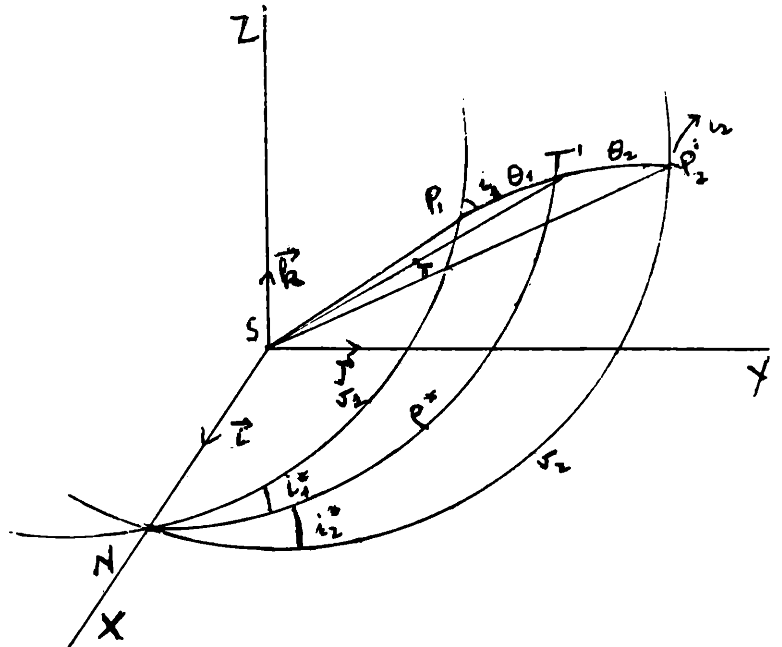


Fig. 3 — The set XYZ is centered in the Sun, the X axis being nodal line of the orbital planes of P_1 and P_2.

Let T' be the projection of T on the Celestial Sphere. We shall consider the great circle passing through N and T', upon which the auxiliary quantity ϱ^* will be computed. We indicate with i_1^* and i_2^* resp. the variable angles between the great circle and the orbital planes of P_1 and P_2; i_1 and i_2 are resp. the inclinations of the moving plane SP_1P_2 with respect to these same planes. We suppose that approximate values of the orbital elements of both planets are known. Then, variable quantities can be determined in a first approximation by means of keplerian elements of P_1 and P_2.

From spherical triangles $T'NP'_1$ and $T'NP'_2$ we get:

1) a) $\cos\theta_1 = \cos v_1 \cos\varrho^* + \sin v_1 \sin\varrho^* \cos i_1^*$
b) $\cos\theta_2 = \cos v_2 \cos\varrho^* + \sin v_2 \sin\varrho^* \cos i_2^*$

2) a) $\sin i_1^* \sin\varrho^* = \sin i_1 \sin\theta_1$
b) $\sin i_2^* \sin\varrho^* = \sin i_2 \sin\theta_2$

3) a) $\sin\varrho^* \cos i_1^* = \cos\theta_1 \sin v_1 - \sin\theta_1 \cos v_1 \cos(180° - i_1)$
b) $\sin\varrho^* \cos i_2^* = \cos\theta_2 \sin v_2 - \sin\theta_2 \cos v_2 \cos(180° - i_2)$

Angles θ_1 and θ_2 must be expressed in terms of known approximate quantities. We first observe that:

$$\vec{SN} = X \vec{i} \quad , \quad \vec{ST} = \xi \vec{i} + \eta \vec{j} + \varsigma \vec{k}$$

$$\cos \varrho^* = \frac{\xi}{|ST|} \quad , \quad \sin \varrho^* = \frac{a}{|ST|}$$

where:

$$a = \sqrt{\eta^2 + \varsigma^2}$$

The components ξ, η, ς can be got, for instance, in the following form:

$$\xi - x_1 = \lambda \, (x_2 - \xi)$$
$$\eta - y_1 = \lambda \, (y_2 - \eta)$$
$$\varsigma - z_1 = \lambda \, (x_2 - \varsigma)$$

λ is a variable parameer depending on the relationship between P_1T and P_2T. λ is such that:

$$\lambda = \frac{|P_1T|}{|P_2T|} = \frac{r_1 (r_1 - r_2 \cos H)}{\left(r_2^2 \; 1 - \frac{r_1}{r_2} \cos H \right)}$$

(fig. 1) where $H = \sphericalangle \, (r_1 \, , \, r_2)$

Formulae 2a,b, and 3a,b give

$$\operatorname{tg} i_j^* = \frac{\sin \theta_j \sin i_j}{\cos \theta_j \sin v_j + \sin \theta_j \cos v_j \cos i_j} \quad , \quad (j = 1, 2)$$

Inclinations i_1^*, i_2^*, i_1 and i_2 are small in general. Time series of the form

4) $\quad i = i^{(0)} + i^{(1)} + i^{(2)} + \dots \quad , \quad (i = i_1 \, , i_2 \, , i_1^* \, . \, i_2^*)$

can be ordinarily obtained from Taylor's expansions.

Formulae 1a,b can be solved by putting in a first approximation $i_1^* = i_2^* = 0$. Once the i^*'s havembeen computed from formulae (4) new values of θ_1 and θ_2 can be got.

These results can be improved after the integration of the corresponding sets of differential equations.